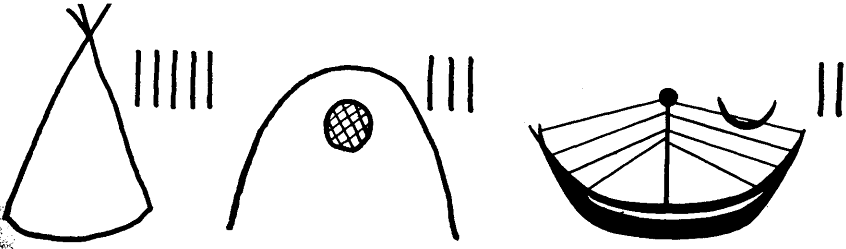
MATHEMATICAL PIE

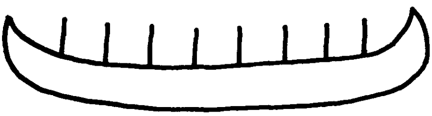
No. 30 — *Editorial Offices :* 97 Chequer Road, Doncaster — MAY 1960

CONCRETE NUMBERS No. 2

The first step in the recording of numbers by our forefathers was dealt with in Part I (May 1959, No. 27) and the kind of pictures mentioned there would have been carved on wood, stone, etc., to record an item of historical interest.

In their endeavours to save time when recording, the primitive races automatically resorted to abbreviations and in the course of time the idea of number as such gradually became clearer and we can guess that the numbers in Part I would have eventually been written as follows :—





N. AMERICAN INDIAN

One of the obvious disadvantages of this symbolism is the amount of space which could be required for the number part of the story.

After this development, the story of writing numbers breaks up into two distinct directions. One part begins to develop into the recording of the spoken language, *i.e.*, the symbolism of writing words, *e.g.*, CUPTIE, whilst the other part deals with the development of the methods of writing " pure numbers," *e.g.*, 3, 5, the symbolism of arithmetic. R.H.C.

SQUARE IN THE FACE

By courtesy of the *Mathematics Students' Journal.*

If x represents a whole number that is also a perfect square, find a formula that will give the next larger square whole number.

76211 01100 44929 32151 60842 44485 96376 69838

MORE ABOUT TRIADS

We can call three numbers a Pythagorean triad if they have no common factor, and the square of one of the numbers is equal to the sum of the squares of the other two. For example, 3, 4, 5 form a Pythagorean triad, but 6, 8, 10 do not. Those of you who read the article on Triads in Pie No. 18 will see that any odd number or any number which is a multiple of 4 can represent one of the perpendicular sides of a right angled triangle whose sides are integers with no common factor.

For example, $11 = 11 \times 1 = (6+5)\ (6-5) = 6^2 - 5^2$

Therefore, squaring both sides, $11^2 = (6^2+5^2)^2 - (2 \times 6 \times 5)^2 = 61^2 - 60^2$
Hence 11, 61, 60 form a Pythagorean triad.

Again, $12 = 2 \times 2 \times 3$ so that $12^2 = 4 \times 2^2 \times 3^2$

Therefore, $12^2 = (2^2+3^2)^2 - (3^2 - 2^2)^2 = 13^2 - 5^2$
so that 12, 13, 5 form another Pythagorian triad.
Also $12 = 2 \times 6 \times 1$. Can you find the other triad to which 12 belongs?

It is not so easy to see what numbers can be the hypotenuse numbers of a Pythagorean triad. The answer is quite simple, although difficult to prove. It is this ; a number can be the hypotenuse number of a Pythagorean triad if, and only if, all its prime factors leave a remainder 1, when divided by 4.

For example, 91 has a prime factor 7, which leaves a remainder 3 when divided by 4. Therefore, 91 is not a hypotenuse number, but
$65 = 5 \times 13$ and $65^2 = 63^2 + 16^2$ and $33^2 + 56^2$
[as well as $(5 \times 5)^2 + (5 \times 12)^2$ and $(13 \times 3)^2 + (13 \times 4)^2$].

There is no simple way of finding the other two members of the triad when the hypotenuse number is given, but, if you are good at algebra, you will be able to do this problem—

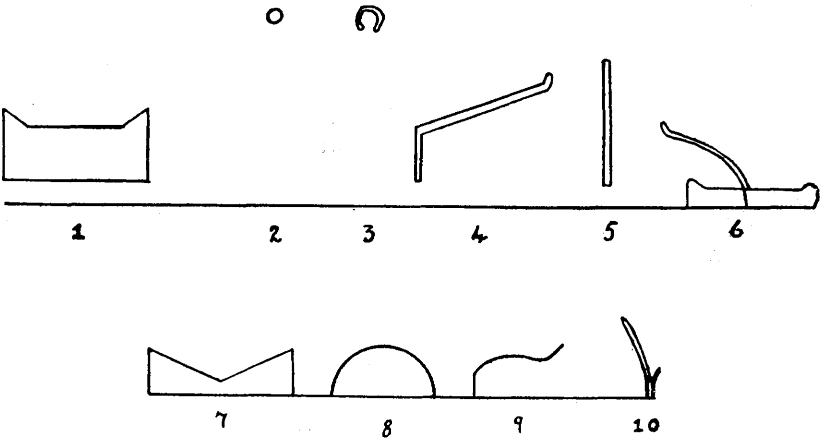
Let $a^2 = b^2 + c^2$ and $p^2 = q^2 + r^2$

Express $(ap)^2$ as the sum of two squares in two different ways. C.V.G.

ROUNDERS

Contributed by H. Bromby, Southampton Grammar School for Girls.

The following shapes are rotated through 360° about the axes shown. Name the familiar objects whose shape they trace out.



95228 68478 31235 52658 21314 49576 85726 24334

STAMP CORNER No. 9

Nature and Nature's laws lay hid in night.
God said "Let Newton be !" and all was light.
ALEXANDER POPE

It did not last, the Devil howling " Ho !
Let Einstein be !" restored the status quo.
SIR JOHN C. SQUIRE

France
18 Franc Blue.

Israel
350 Brown.

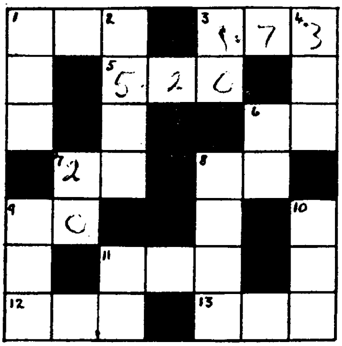
C.V.G.

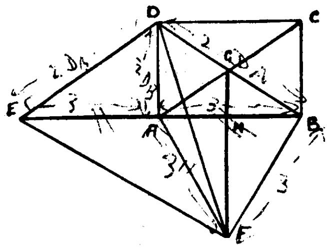
SENIOR CROSS-FIGURE No. 30

Submitted by Mr. W. T. G. Parker, The Grammar School, Minehead.

CLUES ACROSS :

1. The coefficients in order in the quadratic equation whose roots are $-\frac{2}{3}$ and -2.
3. DA.
5. The area of triangle DBF.
6. a times b, where a and b make $ax^3+bx^2+27x+18$ exactly divisible by $(2x+3)$ and by $(x+2)$.
7. $\frac{x}{4}-\frac{y}{2}=3\frac{1}{2}$ and $\frac{x}{9}+2y=16$. Find x.
8. y in 7 across.
9. y varies inversely as the square root of x and $y=30$ when $x=.04$. Find y when $x=.01$.
11. The volume, in cubic inches, of a pyramid on a rectangular base KLMN with vertex V vertically above a point P on KL, where PL=3″, LM=4″, VK=YM=13″.
12. The area of $\triangle$ DAB reversed.
13. The same as 1 across.



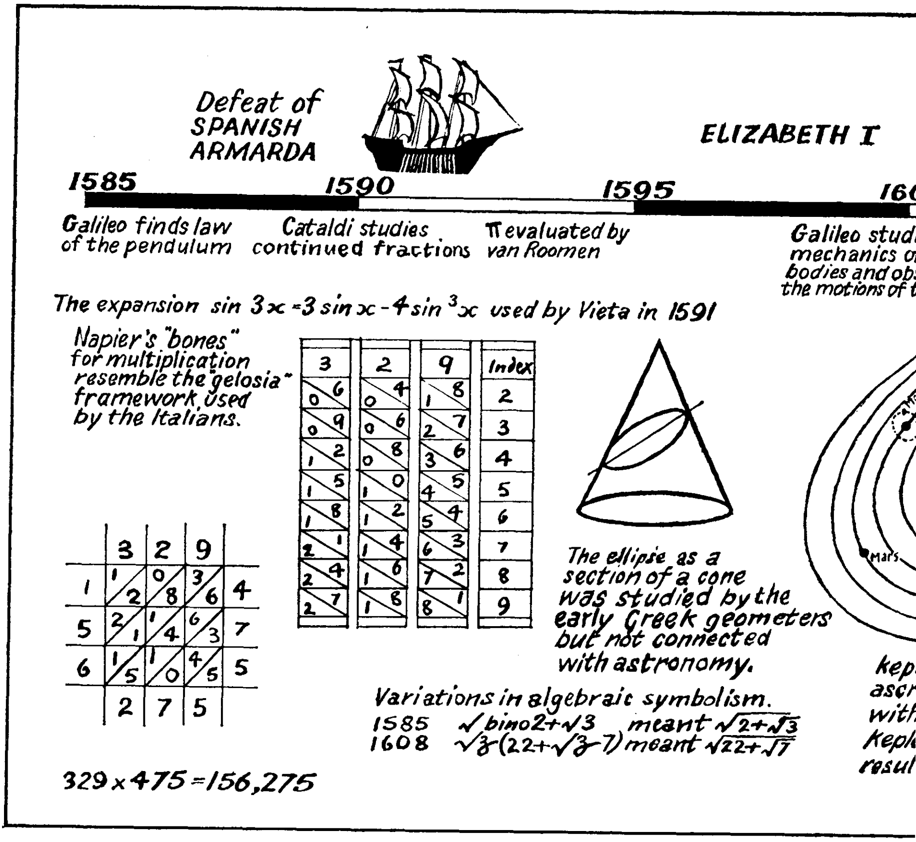


ABCD is a rectangle. AB=3 units, and DB=2DA. AE=AB, and ABF is an equilateral triangle.

CLUES DOWN :

1. The ratio 6.3 : 14.7 : 10.5 expressed as a ratio of smallest whole numbers.
2. DF.
3. The sum of the digits in 2 Down minus the sum of the digits in 4 Down.
4. DE.
6. The maximum value of $2x^3-3x^2-36x-19$.
7. $(2x+9)(2-x)$ when $x=-2$.
8. £10 is borrowed and interest is charged at 3% of the sum owing at the beginning of the year. If £1 is repaid at the end of each year, find, in pounds, the sum owing at the beginning of the sixth year.
9. The area of triangle DFC.
10. The number of tiles, 6 in. square, required to tile the walls of a bathtoom 7 ft. by 9 ft., to a height of 4 ft., the doorway being 3 ft. wide.
11. The height of the pyramid in 11 across, in inches.

41893 03968 64262 43410 77322 69780 28073 18915

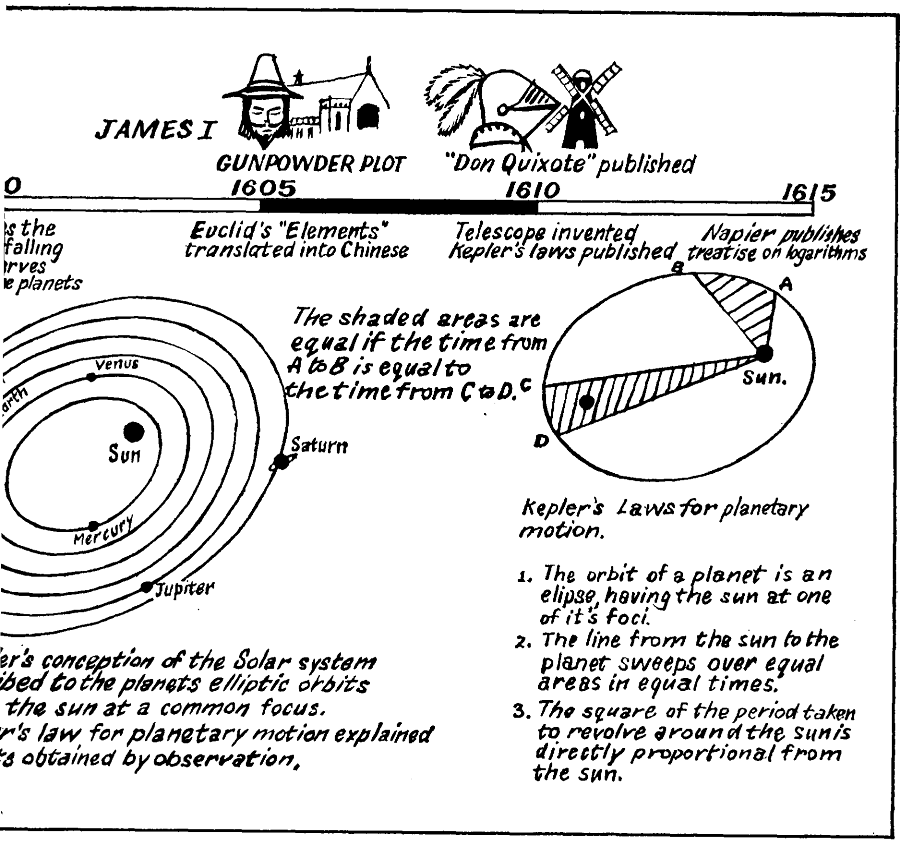


The close of the 16th century and the beginning of the 17th century marked a period in which computational techniques reached a new height; mathematicians were very much in demand and astronomy flourished throughout Europe.

Following closely on the introduction of decimal fractions by Stevin came the invention of logarithms by John Napier. Napier's system of calculation made it possible to reduce multiplication of numbers to the addition of corresponding numbers. The logarithms we use today are based on the same principle as Naperian logarithms but are much more simple in form. As with so many new developments in mathematics, the basic ideas underlying Napier's work were very simple and a number of other mathematicians were inspired to find ways of improving this new tool. Amongst these, two of the most noteworthy contributors were Henry Briggs (1556–1631), a professor of geometry at Gresham College, London, and Edmund Gunter (1581–1626) who is also remembered for his " Gunter's chain " which is used in surveying. Napier's contribution to the task of simplifying calculation came at a time when a great deal of arithmetic was being done in connection with astronomy. Kepler was studying the orbits of the planets, Galileo had begun to use a telescope to study the stars and German mathematicians had constructed trigonometric tables of considerable accuracy. In

44110 10446 82325 27162 01052 65227 21116 60396

HART 8



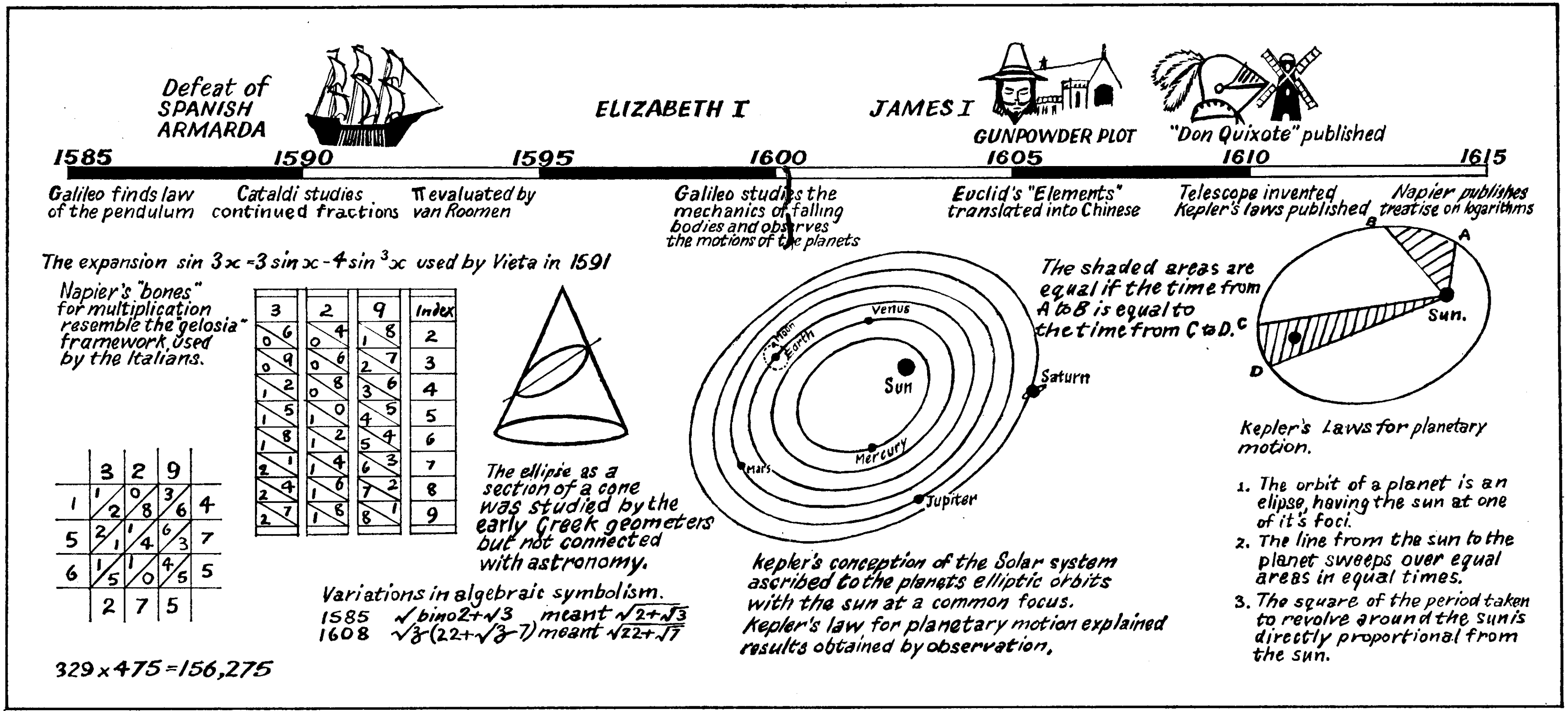
view of this great activity it has been said that the invention of logarithms " by shortening the labours, doubled the life of the astronomer."

The revolution in astronomy due to the works of Copernicus, Tycho Brahe and Johannes Kepler gave man an entirely new vision of his place in the universe and helped to explain many of the phenomena which had puzzled earlier astronomers. Brahe (1546–1601) was a Danish astronomer who spent many years studying the motion of the moon and planets, and the last year of his life was spent in the observatory near Prague with Kepler as his assistant.

Kepler (1571–1630) was more mathematically inclined than Brahe and his first attempt to explain the solar system was made in 1596, when he believed he had discovered a relationship between the five regular solids and the number and distance of the planets. The publication of this theory brought Kepler much fame and at one time he tried to use an oval curve to represent the orbit of Mars. Further reflection brought forth the results which proclaimed his genius and which are now known as " Kepler's Laws." In finding the elliptic orbits of the members of the solar system Kepler bridged the gap between geometry and astronomy of the early Greeks. It has been said " if the Greeks had not cultivated conic sections, Kepler would not have superseded Ptolemy." I.L.C.

66557 30925 47110 55785 37634 66820 65310 98965

TIME CHART 8


Defeat of SPANISH ARMARDA
ELIZABETH I
JAMES I
GUNPOWDER PLOT
"Don Quixote" published
1585
1590
1595
1600
1605
1610
1615
Galileo finds law of the pendulum
Cataldi studies continued fractions
π evaluated by van Roomen
Galileo studies the mechanics of falling bodies and observes the motions of the planets
Euclid's "Elements" translated into Chinese
Telescope invented Kepler's laws published
Napier publishes treatise on logarithms
The expansion sin 3x = 3 sin x - 4 sin ³x used by Vieta in 1591
Napier's "bones" for multiplication resemble the "gelosia" framework used by the Italians.
3
2
9
Index
The ellipse as a section of a cone was studied by the early Greek geometers but not connected with astronomy.
Venus
Moon
Earth
Sun
Saturn
Mars
Mercury
Jupiter
The shaded areas are equal if the time from A to B is equal to the time from C to D.
A
B
C
D
Sun.
Kepler's Laws for planetary motion.
1. The orbit of a planet is an elipse, having the sun at one of it's foci.
2. The line from the sun to the planet sweeps over equal areas in equal times.
3. The square of the period taken to revolve around the sun is directly proportional from the sun.
Kepler's conception of the Solar system ascribed to the planets elliptic orbits with the sun at a common focus.
Kepler's law for planetary motion explained results obtained by observation.
Variations in algebraic symbolism.
1585 √bino2+√3 meant √2+√3
1608 √ȝ(22+√ȝ-7) meant √22+√7
329 x 475 = 156,275

MAGIC SQUARES No. 1

Magic Squares were known to the Chinese over 2,000 years ago, and even today they have a fascination for anyone interested in numbers. In mediaeval times magic squares were used as charms, for it was believed that they had magic powers to ward off the plague and other human ills : this superstition still persists, it is said, in some eastern countries, but their magic for us lies in their peculiar numerical properties. The sum of the numbers in each row, column and diagonal is the same.

8	1	6
3	5	7
4	9	2

Figure 1

Fig. 1 shows the numbers from 1 to 9 arranged in a square with nine cells : if you add the numbers in any row, column or diagonal you will get the same total of 15 every time. You will notice that each corner cell contains an even number, and that the central cell is occupied by the middle number of the sequence 1, 2, 3, 4, **5**, 6, 7, 8, 9. This is the only way in which these numbers can be arranged to form a magic square : 5 must be in the central cell, and starting with the number 2 in any of the four corner squares, the order of the numbers in the cells around the central cell is 27618394, either clockwise or anticlockwise.

Question 1. You can see in Fig. 1 that the numbers in the middle row 3, 5, 7, form a sequence (arithmetical progression) ; what other sequences can you find in the middle column, and the two diagonals ?

15	2	13
8	10	12
7	18	5

Figure 2

Question 2. Fig. 2 shows another magic square made with nine other numbers : what is the magic total of each row, column and diagonal ?

Question 3. Write down the nine numbers in ascending order of magnitude : in which cell is middle number of the series ?

Question 4. What sequences (arithmetical progressions) can you find in the rows, columns or diagonals of Fig. 2 ?

9	1	8
5		

Figure 3

Question 5. It is possible to complete a magic square with nine cells, if the numbers in four cells are given, provided three cells are in a straight line, as the sum of these three numbers will give the magic total. Complete the magic square shown in Fig. 3 and verify that the number in the central cell is the middle number of the nine numbers arranged in order of magnitude, and that there are four sequences (arithmetical progressions)—in the middle row, middle column, and in both diagonals.

5		9
	6	

Figure 4

Question 6. Remembering that the numbers in each diagonal form sequences, complete the magic square indicated by Fig. 4.

Question 7. In Fig. 5 put any number you like in the central cell : (to avoid negative numbers, the number chosen should not be less than 7). Now complete the magic square, using the

26918 62056 47693 12570 58635 66201 85581 00729

	2	
		4

Figure 5

sequence principle. If you do this correctly, you will find that one cell always contains the number 6, whatever number you had put in the central cell. Which cell is it?

Question 8. (For those who like algebra). In Fig. 5, put x instead of 2, and y instead of 4, and prove that whatever number you put into the central cell, one of the other cells must contain $2y - x$.

Submitted by Canon Eperson, Bishop Otter School.

(*to be continued*)

CALCULATING PRODIGIES—No. 1.

George Parker Bidder, 1806-1878

George Bidder was one of a number of remarkable calculating prodigies who lived during the 19th century, and amazed audiences all over England with his exceptional powers in mental arithmetic.

The son of a stone-mason, he was born in Moretonhampstead, Devonshire, and, at the age of six, was taught to count up to 100. Using this limited knowledge he taught himself to add, subtract and multiply numbers less than 100 by making patterns with marbles and buttons, though he still remained ignorant of how to write numbers.

At 7 years old he had already gained a reputation in the village for his ability to calculate quickly. During the next two years his fame spread beyond the village and his father found it profitable to take him about the country to give public exhibitions. Finally, his father was persuaded to leave him in the care of some members of the University of Edinburgh and Bidder later graduated and became a civil engineer.

In spite of his very limited knowledge at the beginning of his career, Bidder learned quickly with practice. Most of the questions posed to him during his early years involved the mental addition and multiplication of large numbers, but by 1819 he was able to calculate square roots and cube roots of equally high numbers and could give almost immediate answers to problems on compound interest.

One of the questions posed when he was 9 years old was : " If the moon be distant from the earth 123,256 miles, and sound travels at the rate of 4 miles a minute, how long would it be before the inhabitants of the moon could hear the battle of Waterloo? " In less than one minute he gave the answer : " 21 days, 9 hours, 34 minutes."

At 14 years old he was asked : " Find a number whose cube less 19 multiplied by its cube shall be equal to the cube of 6." The answer 3 was given instantly.

It is worth noting that with all these questions Bidder was quick to grasp the requirements of a spoken problem, but a written question took him much longer to understand, and he never wrote down any part of his calculation.

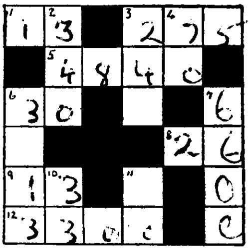
I.L.C.

JUNIOR CROSS-FIGURE No. 27

Submitted by Suzanne Bendz and Sheila Walmark, Hazeldene School, Salcombe, S. Devon

CLUES ACROSS :

1. $3x+7-3y=19$. Find $x+2y$. $2x+y-2=15$.
3. The rate of interest on a loan is increased from 3% to $3\frac{1}{2}$%. The annual interest is raised by £1 7s. 6d. How much is the loan?
5. Number of square yards in an acre.
6. A boy receives 18 marks out of 60. What is his percentage?
8. Solve $x+31+7-8-x+2x=82$
9. Considered unlucky by some.
12. $780+340+60+120+2000$.



CLUES DOWN :

2. The second number in 12 across.
3. CMLXXX ÷ IV.
4. XXVIII+XII+XIV−III−I+XX.
6. 6,404 as a percentage of 18,760 correct to 2 places of decimals.
7. Twice 12 across.
10. Half the total external surface area of a cylinder, closed at one end and open at the other, with external dimensions—radius 3″, height 2″.
11. Solve $3x-10-x=30-2x$.

SOLUTIONS TO PROBLEMS IN ISSUE No. 29

SENIOR CROSS FIGURE No. 29

CLUES ACROSS : (1) 144 ; (4) 211 ; (7) 625 ; (8) 27 ; (9) 999 ; (10) 25 ; (11) 12 ; (13) 4840 ; (14) 16 ; (16) 90 ; (18) 429 ; (19) 81.
CLUES DOWN : (1) 169 ; (2) 429 ; (3) 459 ; (5) 125 ; (6) 17 ; (10) 2240 ; (11) 1899 ; (12) 81 ; (15) 64 ; (17) 21.

GEOMETRY IN A GARDEN.

By using two applications of the circle of Appollonius, it can be shown that the area of the garden is very approximately 3,200 sq. ft.

CARD TRICK.

Place the first card in the centre of the tray. Now wherever the opponent puts a card there will be a space symmetrically opposite in which the first player can put his next card. By continuing in this way, the first player must win.

DO YOU KNOW?

1. Sir Isaac Newton.
2. Izaak Walton.
3. Plato's school.
4. The base angles of an isosceles triangle are equal.
5. Brackets out, divide, multiply, add, subtract. This gives the order for simplifying arithmetic expressions.

PUZZLE CORNER.

Any odd number can be written as $(2n+1)$. Squaring gives $4n^2+4n+1$. Divide by 2, $2n^2+2n+\frac{1}{2}$. The two integers above and below are $2n^2+2n+1$ and $2n^2+2n$. Then

$$(2n^2+2n)^2+(2n+1)^2 = 4n^4+8n^3+8n^2+4n+1 \text{ and}$$
$$(2n^2+2n+1)^2 = 4n^4+8n^3+8n^2+4n+1.$$

MATCH YOUR WITS.

(1) Form a regular tetrahedron with the matches for the edges. (2) The horses cost £$36\frac{4}{11}$ and £$43\frac{7}{11}$

BINARY CROSS-FIGURE.

CLUES ACROSS : (1) 11 ; (100) 1.11 ; (110) 1001 ; (111) 11.
CLUES DOWN : (10) 1100 ; (11) 11 ; (101) 1011 ; (110) 10.

SERIOUSLY SPEAKING.

The next number in the series is 46. The rule for the series is reverse the digits of the perfect squares. The editor regrets the error in printing 36 instead of 63 in the series.

SUM AND PRODUCT.

The product is easier to evaluate as anything multiplied by zero is zero.

The editor apologises for the error in the caption of Cubist Art. It should read $a^3+b^3=(a+b)(a^2-ab+b^2)$.

B.A.

53249 44166 80396 26579 78771 85560 84552 96541

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